Summer 2019 Review
For
Students Entering
All levels of Algebra II and
College Prep Math

Holy Name High School

Directions: Complete the following problems

1) On loose leaf paper

2) Write page number and problem number

3) Use the calculator only to check manual answers

4) Show your work.

5) <u>Due Thursday, August 22, 2019.</u>

(**NOTE:** Use www.khanacademy.org www.mathisfun.com or www.purplemath.com to find specific math related topics with accompanying videos and demonstrations)

NAME: ____________________

Name ______________________ ~~Date~~ Pg. 1

Properties of Square Roots

A **radical expression** is an expression that contains a radical. A radical expression involving square roots is in **simplest form** when these three conditions are met.

- No radicands have perfect square factors other than 1.
- No radicands contain fractions.
- No radicals appear in the denominator of a fraction.

You can use the properties below to simplify radical expressions involving square roots.

Product Property of Square Roots	**Quotient Property of Square Roots**
$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, where $a, b \geq 0$	$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, where $a \geq 0$ and $b > 0$

Example 1 Simplify (a) $\sqrt{75}$ and (b) $\sqrt{\frac{13}{25}}$.

a. $\sqrt{75} = \sqrt{25 \cdot 3}$ Factor using the greatest perfect square factor.

$= \sqrt{25} \cdot \sqrt{3}$ Product Property of Square Roots

$= 5\sqrt{3}$ Simplify.

b. $\sqrt{\frac{13}{25}} = \frac{\sqrt{13}}{\sqrt{25}}$ Quotient Property of Square Roots

$= \frac{\sqrt{13}}{5}$ Simplify.

When a radical is in the denominator of a fraction, you can multiply the fraction by an appropriate form of 1 to eliminate the radical from the denominator. This process is called **rationalizing the denominator**.

Example 2 Simplify $\frac{10}{\sqrt{7}}$ by rationalizing the denominator.

$\frac{10}{\sqrt{7}} = \frac{10}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}}$ Multiply by $\frac{\sqrt{7}}{\sqrt{7}}$.

$= \frac{10\sqrt{7}}{\sqrt{49}}$ Product Property of Square Roots

$= \frac{10\sqrt{7}}{7}$ Simplify.

Practice

Check your answers at BigIdeasMath.com.

Simplify the expression.

1. $\sqrt{12}$ **2.** $-\sqrt{45}$ **3.** $\sqrt{500}$

5. $\sqrt{\frac{3}{4}}$ **7.** $-\sqrt{\frac{8}{25}}$ **8.** $\sqrt{\frac{48}{81}}$

9. $\frac{3}{\sqrt{5}}$ **10.** $-\frac{14}{\sqrt{10}}$

Name ________________________ ~~Date~~ Pg. 2

Powers and Exponents

A **power** is a product of repeated factors. The **base** of a power is the common factor. The **exponent** of a power indicates the number of times the base is used as a factor.

base → $\left(\frac{2}{5}\right)^3$ ← exponent

$$\underbrace{\left(\frac{2}{5}\right)^3}_{\text{power}} = \underbrace{\frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5}}_{\frac{2}{5}\text{ is used as factor 3 times}}$$

Example 1 Write each product using exponents.

a. $(-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9)$

Because -9 is used as a factor 5 times, its exponent is 5.

▶ So, $(-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9) = (-9)^5$.

b. $\pi \cdot \pi \cdot h \cdot h \cdot h$

Because π is used as a factor 2 times, its exponent is 2. Because h is used as a factor 3 times, its exponent is 3.

▶ So, $\pi \cdot \pi \cdot h \cdot h \cdot h = \pi^2 h^3$.

Example 2 Evaluate each expression.

a. $(-5)^4$

$(-5)^4 = (-5) \cdot (-5) \cdot (-5) \cdot (-5)$ Write as repeated multiplication.

$= 625$ Simplify.

b. -5^4

$-5^4 = -(5 \cdot 5 \cdot 5 \cdot 5)$ Write as repeated multiplication.

$= -625$ Simplify.

Practice

Check your answers at BigIdeasMath.com.

Write the product using exponents.

1. $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$
2. $\left(-\frac{1}{3}\right) \cdot \left(-\frac{1}{3}\right) \cdot \left(-\frac{1}{3}\right)$
3. $x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y$
4. $2.5 \cdot 2.5 \cdot b \cdot b \cdot b \cdot b$
5. $(-n) \cdot (-n) \cdot (-n) \cdot (-n)$
6. $(-12) \cdot (-12) \cdot v \cdot v \cdot v$

Evaluate the expression.

7. 10^4
8. -15^2
9. $\left(\frac{3}{4}\right)^3$
10. $\left(-\frac{1}{2}\right)^5$

Name ______________________ Date Pg. 3

Properties of Exponents

Product of Powers	Power of a Product	Power of a Power	
$a^m \cdot a^n = a^{m+n}$ Add exponents.	$(ab)^m = a^m b^m$ Find the power of each factor.	$(a^m)^n = a^{mn}$ Multiply exponents.	
Quotient of Powers	**Power of a Quotient**	**Negative Exponent**	**Zero Exponent**
$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$ Subtract exponents.	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$ Find the power of the numerator and the power of the denominator.	$a^{-n} = \frac{1}{a^n}, a \neq 0$	$a^0 = 1, a \neq 0$

Example 1 **Evaluate (a) 4.9^0 and (b) $(-3)^{-4}$.**

a. $4.9^0 = 1$ Definition of zero exponent

b. $(-3)^{-4} = \frac{1}{(-3)^4}$ Definition of negative exponent

$= \frac{1}{81}$ Evaluate power.

Example 2 **Simplify each expression. Write your answer using only positive exponents.**

a. $2^3 \cdot 2^4 = 2^7 = 128$

b. $\frac{5^9}{5^6} = 5^{9-6} = 5^3 = 125$

c. $\frac{12y^0}{x^{-7}} = 12y^0x^7 = 12x^7$

d. $\frac{x^6 \cdot x^2}{x^5} = \frac{x^{6+2}}{x^5} = x^{8-5} = x^3$

e. $(z^4)^2 = z^{4 \cdot 2} = z^8$

f. $(6mn)^3 = 6^3 \cdot m^3 \cdot n^3 = 216m^3n^3$

g. $\left(\frac{y}{3}\right)^4 = \frac{y^4}{3^4} = \frac{y^4}{81}$

h. $\frac{10x^6y^{-2}}{5x^3y} = \frac{10}{5}x^{(6-3)}y^{(-2-1)} = 2x^3y^{-3} = \frac{2x^3}{y^3}$

Practice

Check your answers at BigIdeasMath.com.

Evaluate the expression.

1. $(-9)^0$ **2.** -8^{-1} **3.** 4^{-3} **4.** $\frac{-5^0}{3^{-2}}$

Simplify the expression. Write your answer using only positive exponents.

5. $2^9 \cdot 2^{-6}$ **6.** $-\frac{10^8}{10^{12}}$ **7.** $y \cdot y^{-5}$ **8.** $\frac{x^7}{x^{-7}}$

9. $-5x^7 \cdot x^{-11} \cdot 2x^4$ **10.** $\frac{x^{-2}}{5z^0}$ **11.** $(w^2)^{-3}$ **12.** $(8xy)^2$

13. $3x^5 \cdot (-2x)^4$ **14.** $(-5m^2n^{-1})^3$ **15.** $\frac{z^8}{z^{-2} \cdot z^9}$ **16.** $\frac{(x^5)^3}{x^6}$

17. $\left(\frac{3x}{2}\right)^3$ **18.** $\left(\frac{6x^4}{5y}\right)^{-2}$ **19.** $\frac{xy^{-2}}{x^4y^{-3}}$ **20.** $\frac{8xy}{6x^5yz^{-2}}$

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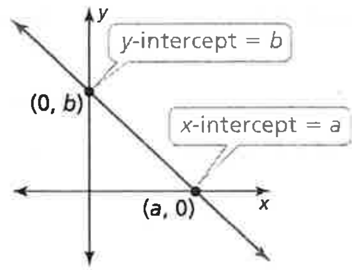
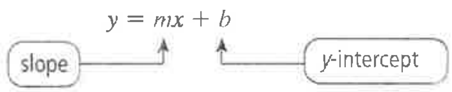
Slope-Intercept Form

The **x-intercept** of a line is the x-coordinate of the point where the line crosses the x-axis. It occurs when $y = 0$.

The **y-intercept** of a line is the y-coordinate of the point where the line crosses the y-axis. It occurs when $x = 0$.

A linear equation written in the form $y = mx + b$ is in **slope-intercept form**. The slope of the line is m, and the y-intercept of the line is b.





Example 1 **Identify the slope and the y-intercept of the graph of each linear equation.**

a. $y = -3x - 8$

$y = -3x + (-8)$ Write in slope-intercept form.

▶ The slope is -3, and the y-intercept is -8.

b. $y - 4 = \frac{1}{3}x$

$y = \frac{1}{3}x + 4$ Add 4 to each side.

▶ The slope is $\frac{1}{3}$, and the y-intercept is 4.

Example 2 **Find the x-intercept and the y-intercept of the graph of $2x + y = 4$.**

To find the x-intercept, substitute 0 for y and solve for x.

$$2x + y = 4$$
$$2x + (0) = 4$$
$$x = 2$$

To find the y-intercept, substitute 0 for x and solve for y.

$$2x + y = 4$$
$$2(0) + y = 4$$
$$y = 4$$

▶ The x-intercept is 2, and the y-intercept is 4.

Practice

Check your answers at BigIdeasMath.com.

Identify the slope and the y-intercept of the graph of the linear equation.

1. $y = 4x + 7$
2. $y = -\frac{1}{3}x + 8$
3. $y = \frac{1}{9}x - 6$

Find the x-intercept and the y-intercept of the graph of the equation.

7. $y = 2x$
8. $y = x + 8$
9. $y = 3x + 6$

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Writing Linear Equations

Given a point on a line and the slope of the line, you can write an equation of the line.

Example 1 **Write an equation in slope-intercept form of the line that passes through the point (−5, 6) and has a slope of $\frac{3}{5}$.**

$y = mx + b$	Write the slope-intercept form.
$6 = \frac{3}{5}(-5) + b$	Substitute $\frac{3}{5}$ for m, −5 for x, and 6 for y.
$6 = -3 + b$	Simplify.
$9 = b$	Solve for b.

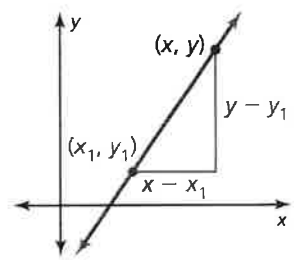
▶ So, the equation is $y = \frac{3}{5}x + 9$.

A linear equation written in the form $y - y_1 = m(x - x_1)$ is in **point-slope form**. The line passes through the point (x_1, y_1), and the slope of the line is m.

slope

$$y - y_1 = m(x - x_1)$$

passes through (x_1, y_1)



Example 2 **Write an equation in point-slope form of the line that passes through the point (−8, 3) and has a slope of $\frac{3}{4}$.**

$y - y_1 = m(x - x_1)$	Write the point-slope form.
$y - 3 = \frac{3}{4}[x - (-8)]$	Substitute $\frac{3}{4}$ for m, −8 for x_1, and 3 for y_1.
$y - 3 = \frac{3}{4}(x + 8)$	Simplify.

▶ So, the equation is $y - 3 = \frac{3}{4}(x + 8)$.

Practice

Check your answers at BigIdeasMath.com.

Write an equation in slope-intercept form of the line that passes through the given point and has the given slope.

1. $(1, 3);\ m = 2$ **2.** $(4, 2);\ m = 3$ **3.** $(-2, 3);\ m = \frac{1}{2}$

Write an equation in point-slope form of the line that passes through the given point and has the given slope.

7. $(1, 1);\ m = 5$ **8.** $(-3, 4);\ m = 2$ **9.** $(6, -3);\ m = \frac{3}{2}$

Name ______________________ ~~Date~~ Pg. 6

Solving Systems of Equations

A **system of linear equations** is a set of two or more linear equations in the same variables. An example is shown at the right. A **solution of a system of linear equations** in two variables is an ordered pair that is a solution of each equation in the system.

$x + 2y = 5$ Equation 1
$x - y = -1$ Equation 2

Example 1 Solve the system of linear equations above by (a) graphing, (b) substitution, and (c) elimination.

a. Graph each equation. The graphs appear to intersect at (1, 2). Check this point.

Equation 1 $x + 2y = 5$

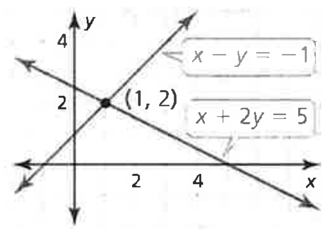
$1 + 2(2) \stackrel{?}{=} 5$

$5 = 5$ ✓

Equation 2 $x - y = -1$

$1 - 2 \stackrel{?}{=} -1$

$-1 = -1$ ✓



▶ The solution is (1, 2).

b. Solve for x in Equation 2.

$$x - y = -1$$
$$x = y - 1$$

Substitute $y - 1$ for x in Equation 1 and solve for y.

$$x + 2y = 5$$
$$(y - 1) + 2y = 5$$
$$y = 2$$

Substitute 2 for y in Equation 2 and solve for x.

$$x - y = -1$$
$$x - 2 = -1$$
$$x = 1$$

▶ The solution is (1, 2).

c. Multiply Equation 2 by 2. Then add the equations and solve the resulting equation.

$$x + 2y = 5 \Rightarrow x + 2y = 5$$
$$x - y = -1 \Rightarrow \underline{2x - 2y = -2}$$
$$3x = 3$$
$$x = 1$$

Substitute 1 for x in Equation 2 and solve for y.

$$x - y = -1$$
$$1 - y = -1$$
$$2 = y$$

▶ The solution is (1, 2).

Practice

Check your answers at BigIdeasMath.com.

Solve the system of linear equations by graphing.

1. $y = x - 3$
$y = -x + 1$

3. $2x + y = 5$
$4x - 2y = 6$

4. $9x - 3y = 3$
$3x + y = 1$

Solve the system of linear equations by substitution.

5. $y = 1 - x$
$-2x + y = 4$

7. $3x - y = 5$
$2x - y = -3$

8. $x - 2y = -3$
$7x - 2y = 15$

Solve the system of linear equations by elimination.

9. $-2x + 2y = -2$
$2x + y = 5$

11. $x + 5y = -2$
$5x + y = 14$

12. $2x + 3y = 5$
$4x + 6y = -10$

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Functions

A **relation** pairs inputs with outputs. When a relation is given as ordered pairs, the x-coordinates are inputs and the y-coordinates are outputs. A relation that pairs each input with *exactly one* output is a **function**.

Example 1 **Determine whether each relation is a function. Explain.**

a. $(-1, 3), (0, 3), (1, 3), (2, 1), (3, 1)$

Every input has exactly one output.

▶ So, the relation is a function.

b. $(5, 1), (9, 8), (7, 5), (5, 4), (6, 3)$

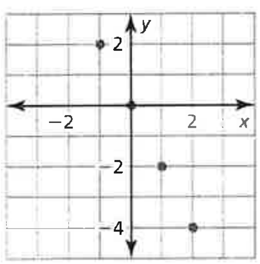
The input 5 has two outputs, 1 and 4.

▶ So, the relation is *not* a function.

The **domain** of a function is the set of all possible input values. The **range** of a function is the set of all possible output values.

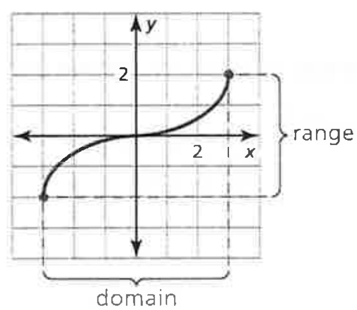
Example 2 **Find the domain and range of the function represented by the graph.**

a.



▶ The domain is -1, 0, 1, and 2.
The range is -4, -2, 0, and 2.

b.



▶ The domain is $-3 \le x \le 3$.
The range is $-2 \le y \le 2$.

Practice

Check your answers at BigIdeasMath.com.

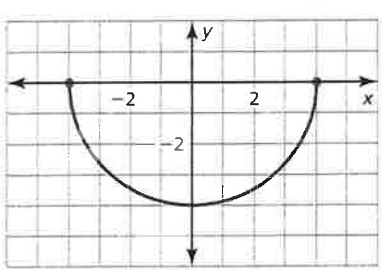
Determine whether the relation is a function. Explain.

1. $(2, -5), (3, -1), (4, 2), (5, -5), (6, 7)$

2. $(8, 5), (6, 0), (4, -7), (2, -4), (4, 7)$

Find the domain and range of the function represented by the graph.

3.



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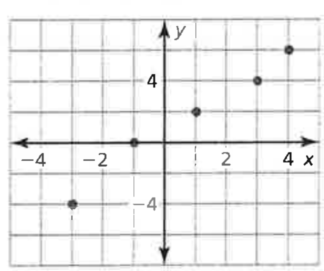

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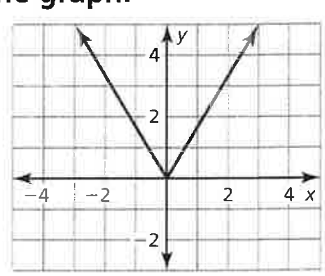

Name ______________________ ~~Date~~ Pg. 8

Function Notation

A linear function can be written in the form $y = mx - b$. By naming a linear function f, you can also write the function using **function notation**.

$$f(x) = mx + b \qquad \text{Function notation}$$

The notation $f(x)$ is another name for y. If f is a function, and x is in its domain, then $f(x)$ represents the output of f corresponding to the input x. You can use letters other than f to name a function, such as g or h.

Example 1 Evaluate the function for the given value of *x*.

a. $f(x) = 2x + 5; x = 7$

$f(7) = 2(7) + 5$ Substitute 7 for x.

$= 14 + 5$ Multiply.

$= 19$ Add.

▶ When $x = 7, f(x) = 19$.

b. $g(x) = 4x - x^2; x = -3$

$g(-3) = 4(-3) - (-3)^2$ Substitute −3 for x.

$= -12 - 9$ Multiply.

$= -21$ Subtract.

▶ When $x = -3$, $g(x) = -21$.

Example 2 Determine whether the ordered pair is a solution of the equation.

a. $h(x) = 8 + x; (-6, 2)$

$2 \stackrel{?}{=} 8 + (-6)$ Substitute −6 for x and 2 for $h(x)$.

$2 = 2$ ✓ Add.

▶ So, (−6, 2) is a solution.

b. $p(x) = |3x - 1|; (-2, -7)$

$-7 \stackrel{?}{=} |3(-2) - 1|$ Substitute −2 for x and −7 for $p(x)$.

$-7 \stackrel{?}{=} |-7|$ Evaluate.

$-7 \neq 7$ ✗ Evaluate.

▶ So, (−2, −7) is *not* a solution.

Practice

Check your answers at BigIdeasMath.com.

Evaluate the function for the given value of *x*.

1. $f(x) = x + 9; x = 8$

3. $h(x) = 4x + 3; x = 10$

4. $n(x) = -x - 4; x = -2$

5. $p(x) = -\frac{3}{4}x^2; x = 6$

7. $k(x) = x^2 + 7x - 1; x = -3$

8. $h(x) = |3x - 8|; x = 1$

Determine whether the ordered pair is a solution of the equation.

10. $f(x) = 3x + 5; (-1, 2)$

13. $n(x) = x^2 - 6x - 1; (4, -7)$

14. $h(x) = |x| - 14; (-4, 10)$

15. $p(x) = |-9x - 2|; (0, 2)$